Appendix C. Relationship Between the Mean and Variability of Revenue and the Price-Yield Correlation

This appendix examines the impact of the correlation between price and yield on revenue. Correlation measures the strength and direction of the linear relationship between two variables. The Pearson correlation in particular can take on values from -1 to 1, and is a measure of the relationship between two random variables. A correlation of -1 means that the two variables move in opposite directions in a perfectly linear fashion (i.e., the movements track along a straight line), and a correlation of 1 means that the two variables move in the same direction in a perfectly linear fashion A correlation of 0 means that there is no relationship between the variables. The relationship gets stronger as the correlation moves from a value of 0 toward -1 or 1. One would expect the price-yield correlation to be 0 or less for crops, with the values varying across crops.

Analysis of national average corn yield and price over 1975 to 2005 suggests that the correlation between these two variables is -0.71 using the statistical approaches discussed in Cooper (2009b, 2007). The correlation between local corn yields and price will tend to be less negative than at the national level, but still less than zero. For example, in Logan County, Illinois, the correlation between county yield and national price is estimated to be -0.68. The value of the natural hedge can be relatively low for counties outside the major producing regions. For example, in Barnes County, North Dakota, the correlation between county yield and national price is relatively low at -0.21. In general, price-yield correlations at the farm level are likely to be lower than at the county level, but to the extent that farm-level yields are correlated with aggregate yield for the region, the price-yield correlation for a farmer in Logan County, Illinois is likely to be higher than for one in Barnes County, North Dakota.

What is the implication for revenue of a nonzero correlation between price and yield? This correlation affects both the mean and variability of revenue. The main text focused on the effect of the natural hedge (the negative correlation) in stabilizing revenue (that is, decreasing the variability of revenue). That the variability of revenue decreases the more negative the correlation is between price and yield can be demonstrated by the statistical formula for the variability of revenue (e.g., Goodman, 1960), but the complexity of this formula is beyond the scope of this report.

What is not generally part of the public discussion of the natural hedge and its implications for revenue is that the more negative the correlation between price and yield, the lower the mean value of revenue. Say that price per bushel = P and yield per acre = Y. Using the formula for the expected value product of two correlated random variables (Mood and Graybill, 1963), the expected value (or mean) of revenue per acre R, which is P times Y, is

(C.1) $E[R] = E[P] \cdot E[Y] + COV(P,Y),$

where $E[P]$ is the expected value of P, and $E[Y]$ is the expected value of Y. $COV(P,Y)$ is a measure of the statistical relationship (covariance) between P and Y and equal to the correlation (P,Y) times the standard deviation of P times the standard deviation of Y. The correlation is essentially a covariance that has been adjusted to fall between -1 and 1. Equation C.1 shows that the more negative the *COV(P,Y)*, the lower the expected revenue, all else being equal. Note that no current or proposed revenue-based commodity support plans include the covariance term in the calculation of expected or target revenue. Doing so would likely lower the probability of a payment being made.

If one is to fix revenue, R, at a commodity revenue coverage level, R_Z, then we can define the combinations of P and Y that will yield the revenue level R_Z, or

$$\text{(C.2)}\quad E[P]=\frac{R_Z - COV(P,Y)}{E[Y]}$$

This function (known as an iso-revenue line) identifies a curve for which, given an expectation of yield, the required price is determined so that the stated revenue R_Z is met (fig. C.1). Say that R_Z is the revenue guarantee to be provided by a revenue support program, and that any actual price-yield combination that produces a revenue lower than R_Z will trigger a support payment that covers the difference. The price-yield combinations that will trigger a support payment are those below the curves in the figure.

Figure C.1 demonstrates the significance of the statistical relationship between price and yield—as defined by the covariance between P and Y—to meeting a given level of revenue. The lower line is the combination of prices and yields that gives the revenue value R_Z when there is no statistical relationship between P and Y (the covariance and the correlation are zero). When the correlation between P and Y is less than 0, the curve moves up, as in the case for the correlation of -1 in the figure.

The more negative the correlation between P and Y, for any given value of yield, the farmer with the more negative price-yield correlation will need a higher price to attain the revenue R_Z, all else being equal.[1]

In summary, there is clearly a tradeoff when it comes to the impact on producers of the natural hedge between price and yield: increasing the magnitude of the natural hedge lowers the mean value of revenue, but it also lowers the variability of revenue. Producer preference for accepting lower mean revenue in exchange for lower revenue variability is discussed in “Producer Preferences for Mean Versus Variability of Gross Revenue.”

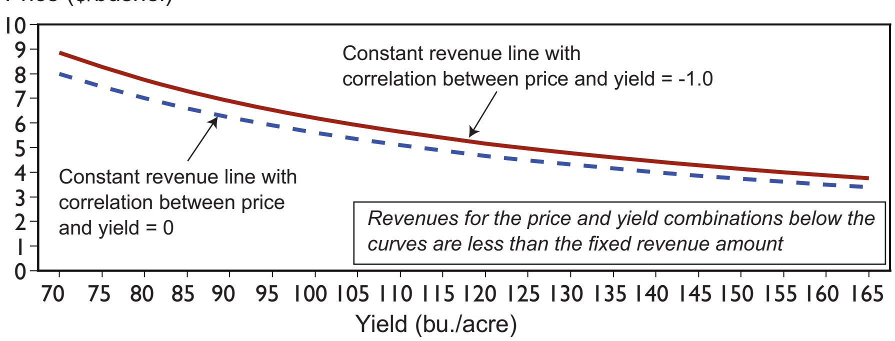
[1]Before the reader is tempted to draw some implications for regional differences in revenue from figure c.1, note that the only difference between the lines in the figure is the covariance between price and yield—the lines are the same in mean price, mean yield, and the standard deviations of price and yield.

Figure C.1

Price-yield relationship over which revenue is constant (hypothetical crop)

The curve shifts upward as the correlation between price and yield becomes more negative

Price ($/bushel)



Yield (bu./acre)

Note: The revenue for both lines is fixed at $560, and the standard deviation of price and yield are fixed at 1.5 and 40, respectively.